SYLLABUS FOR POST GRADUATE MATHEMATICS

Semester-I

ALGEBRA (MAT 215111)

Total Marks: 50   Credit Points: 4   Duration: 6 months

1. **Group Theory**

   Group action. Permutation representation and Caley's theorem.
   Conjugacy classes and class equation, p-groups.
   Converse of Lagrange's theorem for finite abelian groups.
   Sylow's theorems and its applications.
   Direct product, finitely generated abelian groups.
   Solvable groups- solvability of S_n.

2. **Ring Theory**

   Subrings and ideals, principal ideals, Principal Ideal Domain (PID).
   Quotient ring, isomorphism and correspondence theorems.
   Prime, primary and maximal ideals - examples, characterizations and their interrelations.
   Divisor, common divisor and greatest common divisor; prime and irreducible elements, characterizations of prime and maximal ideals in terms of prime and irreducible elements.
   Factorization Domain (FD) and Unique Factorization Domain (UFD).
   Ring with chain conditions - Noetherian rings and Artenian rings (definition only).
   Polynomial Rings.

3. **Field Extension and Galois Theory**

   Field extension-algebraic and transcendental extension and their characterizations.
   Splitting field and algebraic closure. Separable and normal extension. Cyclotomic polynomial and Galois field.
   Galois theory - introduction. Basic ideas and results focusing the fundamental theorem of Galois theory.
   Solvability by radicals.

References

1. *D. S. Malik, John M. Mordeson and M. K. Sen*, Fundamentals of Abstract Algebra, McGraw Hill, International Edition, 1991
2. *Dummit and Foote*, Abstract Algebra, John Wiley and Sons Inc.
3. *I. H. Hungerford*, Algebra, Springer Verlag
4. *John B. Fraleigh*, A First Course in Abstract Algebra, Narosa Publishing House
5. *I. N. Herstein*, Topics in Algebra, Wiley Eastern Limited, New Delhi, 1975
6. *S. Lang*, Abstract Algebra, 2^{nd} edition, Addison-Wesley

LINEAR ALGEBRA (MAT 215112)

Total Marks: 50   Credit Points: 4   Duration: 6 months

Modules, Submodules, Quotient Modules, Morphisms, Isomorphism Theorems, Correspondence Theorem, Simple Modules, Free modules, Noetherian and Artinian Modules, Exact Sequence, Dual Modules, Fundamental Structure Theorem for Finitely Generated Modules over PID (Statement only).

Matrices and Linear Transformations, Representation of Linear Transformations by matrices, Rank-Nullity Theorem, Linear Functionals, Dual Spaces, Dual basis.

Matric Polynomial, Minimal Polynomial, Characteristic Polynomials and Characteristic Roots. Diagonalization of Matrices. Reduction to Triangular Forms. Jordan Blocks.

Jordan Canonical Forms, Invariant Factors, Rational Canonical Forms, Smith Normal Form over a PID.

Bilinear Forms, Quadratic Forms

References

1. *M. Artin*, Algebra, Prentice Hall of India, 1994.
2. *K. Hoffman and R. Kunze*, Linear Algebra, Pearson Education (India), 2003. Prentice-Hall of India, 1991.
3. *S. Lang*, Linear Algebra, Undergraduate Texts in Mathematics, Springer Verlag. New York, 1989.
4. *A.R. Rao. P. Bhimasankaram*, Linear Algebra, Tata-McGraw Hill.
5. *P. Lax*, Linear Algebra, John Wiley & Sons, New York. Indian Edition, 1997
6. *H. E. Rose*, Linear Algebra, Birkhauser, 2002
7. *S. Lang*, Algebra, 3^{rd} edition, Springer (India), 2004
8. *G. Strang* Linear Algebra & its Applications. Harcourt Brace Jovanovich 3^{rd} edition 1998.
9. *B. Noble and J.W. Daniel*, Applied Linear Algebra, Prentice Hall, NJ, 3^{rd} edition, 1988.
10. *N.J. Pullman*, Matrix Theory and its Applications, Marcel Dekker Inc. New York, 1976.
11. *I. N. Herstein*, Topics in Algebra, Wiley Eastern Limited, New Delhi, 1975
12. *R. Stall*, Linear Algebra and Matrix Theory
13. *Evar D. Nering*, Linear Algebra and Matrix Theory
14. *B. C. Chatterjee*, Linear Algebra
15. *Rudra Pratap*, Getting Started with MATLAB 7, Oxford Press, Indian edition, 2007.

REAL ANALYSIS (MAT 215113)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Lebesgue measure: Definition of Lebesgue outer measure and inner measure. Measurable functions : Definition on a measurable set in $\mathbb{R}$ and basic properties. Simple functions.

Functions of bounded variation: Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability.

Absolutely continuous functions: Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation; Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere.

Riernann-Stieltjes intcgral: Existence and basic properties, Integration by parts, Integration of a continuous function with respect to a step function, Convergence theorems in respect of integrand.

Differentiation on $\mathbb{R}^n$: Directional derivatives and continuity, the total derivative and continuity, total derivative in terms of partial derivatives, the matrix transformation of $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$. The Jacobian matrix.

The chain rule and its matrix form. Mean value theorem for vector valued function. Mean value inequality. A sufficient condition for differentiability. A sufficient condition for equality of mixed partial derivatives. Functions with nonzero Jacobian determinant. The inverse function theorem. The implicit function theorem as an application of inverse function theorem.

Extremum problems with side conditions. Lagrange's necessary conditions as an application of inverse function theorem.

Integration on $\mathbb{R}^n$: Integral of $f : A \longrightarrow \mathbb{R}$ where $A \subseteq \mathbb{R}^n$ is a closed rectangle. Conditions of integrability. Integral of $f : C \longrightarrow \mathbb{R}$ where $C \subseteq \mathbb{R}^n$ is a not a rectangle. Concept of Jordan measurability of a set in $\mathbb{R}^n$. Fubini's theorem for integral of $f : A \times B \longrightarrow \mathbb{R}$, where $A \subseteq \mathbb{R}^n$, $B \subseteq \mathbb{R}^m$ are closed rectangles. Fubini's theorem for integral of $f : C \longrightarrow \mathbb{R}$, where $C \subseteq A \times B$. Formula for change of variables in an integral in $\mathbb{R}^n$.

References

1. *I. M. Apostol*, Mathematical Analysis.
2. *M. Spivak*, Calculus on Manifolds.

COMPLEX ANALYSIS (MAT 215114)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Riemann's sphere, point at infinity and the extended complex plane.

Functions of a complex variable, limit and continuity. Analytic functions. Cauchy-Riemann equations. Complex integration, Cauchy's fundamental theorem statement only) and its consequences. Cauchy's integral formula.

Derivative of an analytic function. Morera's theorem. Cauchy's inequality. Liouville's theorem. Fundamental theorem of classical algebra.

Uniformly convergent series of analytic functions. Power series. Taylor's theorem. Laurent's theorem.

Zeros of an analytic function. Singularities and their classification. Limit points of zeros and poles. Riemann's theorem. Weierstrass-Casorati theorem. Theory of residues. Argument principle. Rouche's theorem. Maximum modulus theorem. Schwarz lemma. Behaviour of a function at the point at infinity.

Contour integration. Conformal mapping. Bilinear transformation. Idea of analytic continuation. Multi valued functions-branch point. Idea of winding number.

References

1. *A I. Markushevich*: Theory of Functions of a Complex variable (Vol. I, II and III).
2. *R. V. Churchill and J. W. Brown*: Complex Variables and Applications.
3. *E. C. Titchmarsh*: The Theory of Functions.
4. *E.T. Copson*: An Introduction to the Theory of Functions of a Complex Variable.
5. *J. B. Conway*: Functions of One Complex Variable.
6. *L. V. Ahlfors*: Complex Analysis.
7. *H. S. Kasana*: Complex Variables-Theory and Applications.
8. *Shantinarayan and P. K Mittal*: Theory of Functions of a Complex Variable.
9. *A.K. Mukhopadhyay*: Functions of Complex Variables and Conformal Transformation.
10. *J. M. Howie* : Complex Analysis

MECHANICS (MAT 215115)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Newton's Laws of Motion. Single and many particle systems. Concepts of energy, momentum, angular momentum, conservation laws.

The Lagrangian formulation: Constraints, generalized coordinates, holonomic and nonholonomic constraints, Lagrange's equations. The Hamiltonian, Noether's Theorem, Hamilton's Equations.

Small oscillations and stability.

Motion of rigid bodies. Euler's Theorem. Inertia tensor, Euler angles, Euler's equations .Torque free motion. Motion of a free top. Motion of a heavy symmetrical top

Hamilton's formulation: Hamilton's principle, Hamilton's equations, Legendre transformation, some conservation laws, Liouville's Theorem, Poisson brackets, canonical transformations, generating functions, action -angle variables, Hamilton Jacobi equation.

The Special Theory of Relativity. Galilean transform. Length contraction. Time dilation. Lorentz transform and consequences.

References

1. *H. Goldstein*: Classical Mechanics
2. *L. Landau and E. Lifshitz*: Mechanics
3. *Rana and Jog*: Classical Mechanics
4. *Fowles and Cassiday*: Analytical Mechanics

COMPUTER PROGRAMMING IN C (MAT 215117)

Total Marks: 25 Credit Points: 2 Duration: 6 months

PRACTICAL

C character set, data types, operators and expressions, input and output operators, control statements, functions, arrays - single and multi-dimensional, strings, pointers, structures and unions, data files. Scope of the variable.

Stacks, queues and linked lists. Sorting and searching algorithms.

References

1. *Byron S. Gottfried*: Programming with C
2. *Brian W. Kernighan and Dennis M. Ritchie*: The C Programming Language
3. *E Balagurusamy*: Programming in ANSI C

Semester-II

TOPOLOGY (MAT 215121)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Brief Description of

Countable and uncountable sets. Axiom of choice and its equivalence. Cardinal and ordinal numbers. Schroeder-Bernstein theorem. Continuum hypothesis. Zorn's lemma and well-ordering theorem.

Fundamentals of Topological spaces

Definition and examples; open and closed sets. Bases and sub-bases. Closure and interior- their properties and relations; exterior, boundary, accumulation points, derived sets, dense set, G_δ and F_σ sets. Neighbourhoods and neighbourhood system. Subspace and induced/relative topology. Relation of closure, interior, accumulation points etc. between the whole space and the subspace.
Alternative methods of defining a topology in terms of Kuratowski closure operator, interior operator, neighbourhood systems.
Continuous, open and closed maps, pasting lemma, homeomorphism and topological properties.

Countability axioms

1^{st} and 2^{nd} countability axioms, Separability, Lindelöffness. Characterizations of accumulation points, closed sets, open sets in a 1^{st} countable space w.r.t. sequences. Heine's continuity criterion.

Separation Axioms

T_i spaces $\left(i = 0, 1, 2, 3, 3\frac{1}{2}, 4, 5\right)$, their characterizations and basic properties.
Urysohn's lemma and Tietze' extension theorem (statement only) and their applications.

Connectedness

Connected and disconnected spaces. Connectedness on the real line. Components and quasi components. Local connectedness.

Compactness

Compactness and some of its basic properties. Compactness and FIP. Continuous functions and compact sets. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Product spaces

Product and box topology, Projection maps. Alexander subbase theorem and Tychonoff product theorem. Separation and product spaces. Connectedness and product spaces.Countability and product spaces.

References

1. *J. L. Kelley*: General Topology, Van Nostrand
2. *S. Willard*: General Topology, Addison-Wesley
3. *J. Dugundji*: Topology, Allyn and Bacon
4. *J. Munkres*: Topology, A first course, Prentice Hall, India
5. *G. F. Simmons*: Introduction to Topology and Modern Analysis, McGraw Hill
6. *K. D. Joshi*: Introduction to General Topology, Wiley Eastern Ltd.
7. *R. Engelking*: General Topology, Polish Scientific Publishers, Warszawa
8. *L. Steen and J. Seebach*: Counter examples in Topology
9. *B. C. Chatterjee, S. Ganguly and M. Adhikari*: A text book of Topology, Asian Books Pvt.

LEBESGUE MEASURE AND INTEGRATION (MAT 215122)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Outer Lebesgue Measure on $\mathbb{R}$ (starting with the concept of length of an Interval). The properties of outer Lebesgue measure m^*.
Outer measure μ^* on S, where S is a space; the concept of μ-measurable sets with the help of μ^*. Necessary and sufficient condition for μ-measurability.
Properties of μ-measurable sets. The structure of μ-measurable sets. The concept of σ-algebra; the σ-algebra of Lebesgue measurable sets.
Properties of Lebesgue measure. Vitali's theorem. The existence of an non-measurable set. The Borel sets and Lebesgue measurable sets-a comparison.
μ-measurable functions, their properties, characteristic functions; step functions.

Lebesgue Integration

The following theorems: i) Let f be a nonnegative measurable function. For each set $E \in \mathcal{M}$ ($\mathcal{M}$ is the σ-algebra of μ-measurable sets) define $v_f(E) = \int_E f$. Then the set function v_f is countably additive on $\mathcal{M}$. ii) Lebesgue's monotone convergence theorem. iii) Fatou's lemma. iv) the theorem on dominated summability v) Lebesgue's dominated convergence theorem.
Necessary and Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration.
The concept of L^p-spaces. Inequalities of Holder and Minkowski. Completion of L^p-spaces.
Convergence in measure, Almost Uniform Convergence, Pointwise Convergence a.e., Convergence Diagrams, counter examples. Egoroff theorem.
Lebesgue Integral in the plane. Product σ-algebra. Product Measure. Fubini's Theorem.

If time permits: Signed Measure and Hahn Decomposition. Jordan Decomposition. Radon-Nikodym theorem.

References

1. *P.R. Halmos*: Measure Theory, Von Nostrand, New York, 1950.
2. *E. Hewitt and K. Stromberg*: Real and Abstract Analysis. Third edition, Springer-Verlag, Heidelber and New York, 1975.
3. *G.D. Barra*: Measure Theory and Integration, Wiley Eastern limited, 1987.
4. *W. Rudin*: Real and Complex Analysis, Tata McGraw-Hill, New York, 1987
5. *I. K. Rana*: An introduction to Measure and Integration, Narosa Publishing House, 1997.
6. *H. L. Royden*: Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.
7. *J. F. Randolf*: Basic Real and Abstract Analysis, Academic Press, New York, 1968.
8. *C. D. Aliprantis and Owen Burkinshaw*: Principles of Real Analysis, Academic Press, 2000.
9. *K. R. Parthsarathy*: Introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
10. *R.B. Bartle*: Elements of Real Analysis

FUNCTIONAL ANALYSIS (MAT 215123)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Metric spaces. Brief discussion on continuity. Completeness, compactness. Hölder and Minkowski inequalities (statement only).

Baire's category theorem. Banach's fixed point theorem. Applications to solutions of certain systems of linear algebraic equations, Fredholm's integral equation of the second kind, implicit function theorem. Kannan's fixed point theorem.

Real and Complex linear spaces. Norm induced metric. Banach spaces. The spaces $\mathbb{R}^n$, $\mathbb{C}^n$, $C[a, b]$, c_{00}, c_0, c and ℓ^p, $(1 \le p \le \infty)$. Riesz's lemma. Finite dimensional normed linear spaces and subspaces. Completeness, compactness criterion, equivalent norms (with topological significance).

Bounded linear operators, various expressions for its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Linear and sublinear functionals. Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces and some of its simple applications.

Conjugate or Dual spaces. Examples. Separability of the Dual space. Reflexive spaces. Examples. Weak and Weak* convergence. Uniform boundedness principle and its simple applications. The Open Mapping Theorem and the Closed Graph Theorem.

Inner Product spaces. Schwarz's inequality. The induced norm. Hilbert spaces. Orthogonality, orthonormality, orthogonal complement. The Riesz representation theorem. Bessel's inequality and its generalisation. Convergence of series corresponding to orthonormal sequence. Fourier coefficient. Parseval's identity. Riesz- Fischer Theorem.

References

1. *W. Rudin*: Functional Analysis
2. *Bachman and Narisi*: Functional Analysis
3. *Kreyszig*: Functional Analysis
4. *G. F. Simmons*: Introduction to Topology and Modern Analysis. McGraw Hill
5. *N. Dunford and H.T.Schwarz*: Linear operators. Part 2,Wiley
6. *A. E. Taylor*: Introduction to Functional Analysis, Wiley
7. *B. V. Limaye*: Functional Analysis, Second Edition. New Age-International Limited, Madras

DIFFERENTIAL EQUATION AND GENERALIZED FUNCTIONS (MAT 215124)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Differential Equations: 25 marks (2 CP)

Uniqueness and existence of solution of the first order initial value problem. Cauchy Peano existence theorem. Lipschitz condition. Picard's method of successive approximations. Picard- Lindeloeff theorem. Continuation of solution. Dependence on parameters and on initial value.

Linear first order systems. Existence and uniqueness of solutions. Fundamental solutions.

The linear ODE of n^{th} order. Existence and uniqueness of solutions. Fundamental solutions and Wronskian. Properties of the Wronskian. The method of variation of parameters.

Boundary value problems of Sturm Liouville type. Solution. Green's function. Properties of Green's function.

Eigenvalue problems. Properties of eigenvalues and eigenfunctions for a Sturm Liouville problem. Fourier expansion in terms of orthonormalised characteristic functions.

Initial value problem and Green's function. Adjoint operators.

Linear ODE in the complex domain. Ordinary points and regular singular points. Series solution. Frobenius' method. Hypergeometric, Legendre and Bessel equations. Hypergeometric functions. Legendre polynomials. Bessel functions of first kind and second kind ..

Generalised Functions: 25 marks (2 CP)

Definition of test functions. Generalised Functions. Regular and Singular. Delta function and Delta convergence. Product and derivative of generalized functions. Slow growth generalized functions.

Fourier Transforms. Fourier series of periodic generalized functions.

References

1. *Coddington and Levinson*: Theory of Ordinary Differential Equations
2. *Birkhoff and Rota*: Ordinary Differential Equations
3. *Ince*: Ordinary Differential Equations
4. *V. S. Vladimiroff*: Generalized Functions in Mathematical Physics

OPERATION RESEARCH AND NUMERICAL ANALYSIS (MAT 215125)

Total Marks: 50   Credit Points: 4   Duration: 6 months

Operations Research: 25 Marks (2 CP)

Linear Programming: A brief review of Simplex Algorithm, Revised Simplex Method, Dual Simplex Method, Computational procedure.

Nonlinear Programming: Local and Global Minima, Gradient vector, Saddle point Problem, Kuhn-Tucker Conditions of Optimality, Quadratic Programming, Methods due to Wolfe and Beale.

Dynamic Programming: Bellman's Principle of Optimality, Recursive equation approach, Single additive constraint-Additively separable return, Single multiplicative constraint- Additively separable return, Single additive constraint and multiplicatively separable return, Solution of LPP by Dynamic Programming.

Integer Programming: Gomory's all Integer Programming Method, Branch and Bound Method.

Inventory Control: Reasons for carrying inventories, Inventory decisions, Concept of EOQ Problem of EOQ with no Shortages and with Shortages.

References

1. *H. A. Taha*: Operations Research, MacMillan Publ., 1982.
2. *Kanti Swarup, P.K.Gupta and Manmohan*: Operations Research, Sultan Chand and Sons, New Delhi
3. *S. S. Rao*: Optimization theory and Application, Wiley Eastern Limited., New Delhi.
4. *G. Hadley*: Nonlinear and Dynamic Programming, Addison-Wesley. 1972.
5. *F. S. Hillier and G.J. Lieberman*: Introduction to Operations Research, McGraw Hill International Edition

Numerical Analysis: 25 marks (2 CP)

Control of round-off-errors, loss of significance, condition and instability.

Solution of Nonlinear Equations: Iterative methods. Convergence of methods, Non-linear system of equations. Newton's method, Quasi-Newton's method, Roots of real polynomial equations, Bairstow's method for quadratic factors, Graeffe's root squaring method.

Numerical Solution of system of linear equations: Triangular factorization methods. Matrix inversion method, Ill conditioned matrix, Power method for extreme eigenvalues and related eigenvectors.

Polynomial Interpolation: Hermite interpolation,Uniqueness, Error term, piecewise polynomial interpolation, Cubic spline interpolation.

Approximation of Functions: Weierstrass's approximation theorem (Statement only), Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomials, Uniform approximation.

Richardson extrapolation. Romberg integration, Runge-Kutta methods, Multistep predictor-corrector methods-Milne's method, Adams-Bashforth method, Adams-Moulton method, Convergence and Stability of numerical methods, finite difference methods for BVPs.

References

1. *S. D. Conte and C. DeBoor*: Elementary Numerical Analysis: An Algorithmic Approach, McGraw Hill, N.Y., 1980.
2. *K.E. Atkinson*: An Introduction to Numerical Analysis, John Wiley and Sons. 1989.
3. *Jain, Iyengar and Jain*: Numerical methods for Scientific and Engineering Computation, New Age International Pub.
4. *F.B.Hilderbrand*: Introduction to Numerical Analysis, Dover Publication.

NUMERICAL ANALYSIS WITH COMPUTER APPLICATION(MAT 215127)

Total Marks: 25 Credit Points: 2 Duration: 6 months

PRACTICAL

Numerical Solution of a System of Equations:

Gauss Elimination and Gauss-Seidel Method for a System of linear Equations

Computation for Finding Inverse Matrix:

L-U Decomposition due to Crout.

Numerical Solution of Algebraic and Transcendental equations:

Newton-Raphson method and Regula -Falsi Method.

Methods for Finding Eigen Pair of a Matrix:

Power Method for Numerically Largest Eigen Value and corresponding Eigen Vector of a Matrix.

Quadrature Rules:

Simpson's one-third rule.

Numerical Solution of ODE:

Runge Kutta Method, Taylor Series Expansion Method, Euler's Formula, Picard's Formula, Milne's Formula

Numerical Solution of PDE:

Finite Difference Method for PDE - Elliptic Type PDE, Parabolic Type PDE, Hyperbolic Type PDE, Crank- Nicholson Scheme for Parabolic Type PDE.

Semester-III

PDE AND INTEGRAL TRANSFORM (MAT 215231)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Partial Differential Equation: 25 Marks (2 CP)

First order partial differential equations - quasi-linear and nonlinear equations.
Higher order equations and characteristics. Classification of second order equations. Wave equation- method of separation of variables, Riemann's method. Laplace equation- Green's method. Heat Conduction equations-boundary value problems. Maximum-minimum principles. Duhamel's principle.

References

1. *Alan Jeffrey*: Applied Partial Differential Equations: An Introduction
2. *Peter V. O'Neil*: Beginning Partial Differential Equations, John Wiley and Sons, 2nd edition.
3. *I.N. Sneddon*: Elements of Partial Differential Equations, McGraw Hill. 1986.
4. *H.F. Weinberger*: A first course in partial differential equations, Blaisdell, 1965.
5. *C.R. Chester*: Techniques in partial differential equations, McGraw Hill, New York, 1971.
6. *K.S. Rao*: Introduction to partial differential equations, Prentice Hall, New Delhi, 1997.
7. *A. Sommerfeld*: Partial differential equations in Physics, Academic Press, New York, 1967.
8. *V. Vladimirov*: Equations of Mathematical Physics. Dekker, New York, 1971.
9. *I. Stakgold*: Green's functions and boundary value problems, Wiley, New York, 1979.

Integral Transforms : 25 Marks (2 CP)

The Fourier Transform: Fourier series, properties of Fourier transform, Inversion formula of Fourier transform, Convolution, Translation, Modulation. Transform of derivatives, Parseval formula, Multiple Fourier transforms, Finite fourier transform, Application to solving boundary value ordinary and partial differential equation.

The Laplace transform: Definition and Properties of Laplace transform, Initial value theorem. Final value theorem. Heaviside expansion theorem. Transform of derivatives. The inversion theorem. Evaluation of inverse transforms by residue. Application to solving P.D.E., lntegral equation etc.

The Z-Transform: Properties of the region convergence of the Z-transform. Inverse Z-transform for discrete-time systems and signals. Signal processing and linear system.

References

1. *I. N. Sneddon*: Fourier Transform, McGraw Hill, 1951.
2. *D. Loknath*: Integral Transforms and their Application, C.R.C. Press, 1995.
3. *E. J. Watson*: Laplace Transforms and Application, Van Nostrand Reinhold Co. Ltd., 1981.
4. *R. V. Churchill*: Operational Mathematics, McGraw Hill, 1958.

DIFFERENTIAL GEOMETRY (MAT 215232)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Differential manifolds, smooth maps and diffeomorphisms, derivatives of smooth maps, Immersion and submersions Submanifolds, Quotient manifolds, Lie groups.

Vector fields, Lie derivative, tangent bundles, vector bundles, flows and exponential maps.

Exterior algebra, differential forms

Orientable manifolds and integration.

Riemannian manifolds, Curvature and parallel transports, geodesic and geodesic completeness, Hopf-Rinow theorem.

References

1. Glen E. Bredon: Topology and Geometry, Springer (GTM), 2005
2. Michael Spivak: Calculus on Manifolds, W.A. Benjamin, 1965
3. F. W. Warner: Foundations of differentiable manifolds and Lie groups, Springer (GTM)
4. S. Helgason: Differential Geometry, Lie groups and symmetric spaces, Springer (GSM)
5. V. Guillemin and A. Pollock: Differential Topology, Englewood and Cliffs
6. J. H. Munkres: Elementary Differential Topology, Princeton University Press, 1963
7. I. M. Singer and J. A. Thorpe: Lecture Notes on Elementary Topology and Geometry, Springer (UGTM)
8. S. Kumaresan: A course in Differential Geometry and Lie Groups, Hindustan Book Agency (TRIM 22)
9. A. Mukherjee: Topics in Differential Topology, Hindustan Book Agency (TRIM 34)

THEORY OF COMPUTATION AND GRAPH THEORY (MAT 215233)

Total Marks: 50 Credit Points: 4 Duration: 6 months

Theory of Computation : 25 marks (2 CP)

Finite Automata -Deterministic and Non-deterministic.
ϵ-moves - Elimination and uses of ϵ-moves, NFA with ϵ-moves, ϵ-closures, Equivalence of NFA and DFA.
Regular Expressions, Regular Languages.
Context Free Grammars and Language, Pushdown Automata, Turing Machines.

References

1. *Bernard M. Moret*: The Theory of Computation
2. *K. L. P. Mishra, N. Chandrasekharan*: Theory of Computer Science
3. *John E Hopcroft, Rajeev Motwani, Jeffery D. Ullman*: Introduction to Automata Theory, Languages and Computation

Graph Theory : 25 marks (2 CP)

Graphs and digraphs. Geometrical representation of graphs. Simple graphs. Null graphs and Regular graphs. Degree of a vertex and degree sequence of a graph. Handshaking lemma due to Euler and some basic properties of a graph. In-degree and out-degree of a vertex in a digraph. Simple digraph and underlying graph. Representation of binary relations on finite sets by digraphs. Reflexive, symmetric and transitive digraphs.
Subgraph, spanning subgraph, induced subgraph on a vertex set and induced subgraph on an edge set. Isomorphism of graphs. Walks, paths, circuits and cycles with their properties. Concatenation of two graphs.
Connected and disconnected graphs. A necessary and sufficient condition for the graph to be disconnected. Components of a graph, decomposition of a graph into a finite number of components,acyclic graph and cycle edge of a graph. Some properties of connected graphs. Complete graphs, disconnecting sets, cut sets, bridge, separating sets and cut vertices, distance between two vertices of a graph. Complement of a graph, Self complimentary graphs, Ramsey problem. Bipartite graphs. Complete bipartite graphs. Necessary and sufficient condition for a simple graph to be bipartite.
Eulerian and Hamiltonian graphs: Euler trials, Euler circuits, Edge traceable graphs. Euler graphs, Euler's Theorem. Fleury's algorithm. Konigsberg bridge problem. Hamiltonian path, Hamiltonian cycle, Hamiltonian graph. A necessary condition for the existence of a Hamiltonian cycle in a connected graph. Sufficient condition for a simple connected graph to be Hamiltonian. Dirac's Theorem, Ore's Theorem (statement only) and its use.
Trees and forests with their properties. Minimally connected graphs, spanning trees. weighted graphs, weight of a spanning tree and minimal spanning trees, Kruskal's algorithm for a minimal spanning tree. The shortest path problem, traveling salesman problem.
Matrix representation of graphs.

References

1. *N. Deo*: Graph Theory with Applications to Engineering and Computer Science, Prentice Half of India, 2000.
2. *C. L.Liu*: Elements of Discrete Mathematics, McGraw-Hill Book Co.
3. *J.P.Tremblay and R.Manohar*: Discrete Mathematical Structures with Applications to Computer Science, Mcgraw-Hill Book Co., 1997.
4. *F. Harary*: Graph Theory, Addison Wesley Publishing Company, 1972.
5. *J. Gross and J. Yellen*: Graph Theory and its Applications, CRC Press(USA), 1999.

MINOR ELECTIVE I (MAT 215234)

Total Marks: 50 Credit Points: 4 Duration: 6 months

1. Continuum Mechanics

Introduction . Idea of continua. Deformation and motion. Axiom of continuity .Continuum motion as a real continuous map.

Analysis of strain. Eulerian and Lagrangian reference frames. Cartesian tensors. Deformation tensor. Strain tensor. Physical interpretation. Displacement and strain. Principal strains. Cauchy strain quadric. Infinitesimal strain and rotation. Compatibility equations.

Analysis of stress. Body forces. Surface forces. Stress vector. Stress tensor.

Balance laws. Equation of continuity. Equation of balance of linear momentum. Stress equations of motion. Equations of balance of angular momentum. Equation of balance of Energy (laws of thermodynamics).

Constitutive equations. Generalized Hooke's law for a solid body. Rate of strain for fluid.

Linear isotropic solids and fluids. Physical interpretation of parameters λ and μ. Equation of motion.

Incompressible viscous fluids. Steady state motion. Equation of motion.

References

1. *Y.C. Fung*: Continuum Mechanics
2. *A.C. Eringen*: Mechanics of Continua
3. *I. S. Sokolnikoff*: Mathematical Theory of Elasticity
4. *Ramsey and Besant*: Treatise on Hydrodynamics
5. *N. E. Kochin*: Theoretical Hydrodynamics

MINOR ELECTIVE I (MAT 215234)

Total Marks: 50   Credit Points: 4   Duration: 6 months

2. Operator Theory and Banach Algebras

Dual spaces, Representation Theorem for bounded linear functionals on $C(a, b]$ and L_p spaces. Dual of $C(a, b]$ and L_p spaces, weak and weak * convergence. Reflexive spaces.

Bounded Linear Operators, Uniqueness Theorem, Adjoint of an Operator and its Properties; Normal, Self Adjoint, Unitary, Projection Operators, their characterizations and properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum and Product of Projections, Invariant Subspaces.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only)

Properties of Bounded Linear Operators, Existence and Representation of the Inverse of I-T Representation of the Resolvent Operators, Non- emptiness of the Resolvent Set, Spectral Radius and its Representation, Spectral Mapping Theorem for Polynomials.

Banach Algebra, Banach Sub Algebra, Identity element, invertible elements, resolvent set and resolvent operator, analytic property of resolvent operator. compactness of spectrum. Division Algebra, Gelfand-Mazur Theorem. Topological divisors of zero. Spectral radius, spectral mapping theorem for polynomial, formula for spectral radius. Complex homomorphism, Gleason-Kahane-Zalazko Theorem, Commutative Banach Algebra, Ideals, maximal ideals, Quotient space as a Banach Algebra under certain conditions. Gelfand theory on representation of Banach Algebra, Gelfand transform. Weak Topology, Weak * Topology, Gelfand Topology, Banach Alaoglu Theorem.

References

1. *W.Rudin*: Functional Analysis
2. *H. Schaefer*: Topological Vector Spaces
3. *E. Kreyszig*: Functional Analysis
4. *G. Bachman and L. Narici*: Functional Analysis
5. *J. Diestel*: Applications of Geometry of Banach Spaces
6. *J. Horvath*: Topological Vector Spaces

MATHEMATICAL MODELLING(MAT 215236)

Total Marks: 25 Credit Points: 2 Duration: 6 months

PRACTICAL

Basic Principles: Modelling using difference/differential equations.

Models in Mathematical Ecology- Single species and two species models.

Traffic Flow Model.

Examples of Mathematical modelling from Mechanics

Stochastic Models: Birth and Death Process

References

1. *J.N. Kapur*: Mathematical Modelling
2. *Neil Gershenfeld*: The Nature of Mathematical Modelling, CUP

Semester-IV

MAJOR ELECTIVE I (MAT 215241)

Total Marks: 50 Credit Points: 4 Duration: 6 months

1. Fuzzy Sets and Their Applications

Unit: 1
Fuzzy sets-Basic definitions. Level sets, Convex fuzzy sets. Basic operations on fuzzy sets. Types of fuzzy sets. Cartesian products. Algebraic products. Bounded sum and difference. f norms and t-co norms.

Unit: 2
The Extension principle-The Zadeh's extension principle image and inverse image of fuzzy sets

Unit: 3
Fuzzy numbers. Elements of fuzzy arithmetic. Fuzzy Relations and Fuzzy Graphs. Fuzzy relations on fuzzy sets. Composition of fuzzy relations, Min-Max composition, and its properties.

Unit: 4
Fuzzy compatibility relations. Fuzzy relation equations. Fuzzy graphs. Similarity relation.

Unit: 5
Fuzzy Logic-An overview of classical logics. Multi valued logics. Fuzzy Propositions. Fuzzy quantifiers. Linguistic variables and hedges.

Unit: 6
Possibility Theory-Fuzzy measures. Evidence theory, Necessity; measure. Possibility theory versus probability theory.

Unit: 7
Decision Making in Fuzzy Environment -individual decision-making. Multi person decision making. Multi criteria decision-making. Multistage decision making fuzzy ranking methods. Fuzzy linear programming.

References

1. *George J. Klir and Tina A. Folger*: Fuzzy sets- Uncertainity and Information, Prentice Hall of India, 1988.
2. *H. J. Zimmerman*: Fuzzy Set theory and its Applications, 4th Edition, Kluwer Academic Publishers, 2001.
3. *George J. Klir and Bo Yuan*: Fuzzy sets and Fuzzy logic: Theory and Applications, Prentice Hall of India, 1997.
4. *Timothy J. Ross*: Fuzzy Logic with Engineering Applications. McGraw Hill International Editions, 1997.
5. *Hung T. Nguyen and Elbert A. Walker*: A First Course in Fuzzy Logic, 2nd edition, Chapman and Hall /CRC 1999.
6. *Jerry M. Mendel*: Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions, PH PTR, 2000.
7. *John Yen and Reza Langari*: Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 1999.

MAJOR ELECTIVE I (MAT 215241)

Total Marks: 50 Credit Points: 4 Duration: 6 months

2. Advanced Topology II

Algebraic Topology : Marks 30

Covering spaces and covering maps-properties and examples. Path lifting and Monodromy theorems, Deck transformation, Van Kampen's theorem (with a discussion of free and amalgamated products of groups), computing fundamental groups via covering spaces.
Singular Homology-Chain complex, Homotopy invariance of a homology, Relation between π_1 and H_1, Relative homology, Exact homology sequence, Excision theorem, Universal coefficient theorem, Kunneth formula, Mayer-Vietoris sequence, Eilenberg-Steenrod axioms for abstract homology theory, Construction of spaces-spherical complexes, cell complexes and more adjunction spaces; Idea of cohomology and cup product.

References

1. *W. S. Massey*: Algebraic Topology
2. *W. S. Massey*: Singular Homology Theory
3. *E. H. Spanier*: Algebraic Topology
4. *B. Gray*: Homotopy Theory- An Introduction to Algebraic Topology
5. *G. E. Bredon*: Geometry and Topology

Rings of Continuous Functions : Marks 20

The ring $C(X)$ and its subring $C^*(X)$, their lattice structure. Ring homomorphism and lattice homomorphism.
Zero-sets. Cozero-sets. Completely separated sets and its characterization, C-embedding and C^*-embedding and their relation, Urysohn's extension theorem. Characterizations of Normal spaces and Pseudocompact spaces in terms of C-embedding and C^*-embedding.
Ideals. Maximal ideals. Prime ideals. Z-ideals, Z-filters, Z-ultra filters, prime filters and their relations. Convergence of Z -filters, cluster points, prime Z -filters and convergence and fixed Z -filters.
Completely regular spaces and the zero-sets. Weak topologies determined by $C(X)$ and $C^*(X)$. Stone-Cech's theorem concerning adequacy of Tychonoff spaces X for investigation of $C(X)$ and $C^*(X)$.
Fixed ideals and compactness, fixed maximal ideals of $C(X)$ and $C^*(X)$, their characterizations.
Structure spaces.

References

1. *Richard E. Chandler*: Hausdorff Compactification, Marcel Dekker, Inc. 1976.
2. *L Gillman and M. Jerison*: Rings of Continuous Functions, Von Nostrand, 1960.
3. *W. J. Thron*: Topological Structures, Halt Reinhart and Winston, 1966.

MAJOR ELECTIVE II (MAT 215242)

Total Marks: 50   Credit Points: 4   Duration: 6 months

1. Fundamentals of Mathematical Biology

1. Single species population dynamics. Linear and nonlinear first order discrete models.Differential equation models. Meta populations. Delay effects. Structured populations. Euler-Lotka equations in discrete and continuous times.

2. Population Dynamics of interacting species.The Lotka-Volterra equations. Modelling the predator functional response. Competition. Interacting meta populations .

3. Infectious Diseases. Simple epidemic and SIS diseases. SIR models.

4. Biological motion. Reaction. Diffusion equations and traveling wave solutions. Spatial spread of epidemics.

5. Pattern formation. Turing instability and bifurcations. Activator inhibitor systems.

6. Tumour modelling.

References

1. *N. Britton*: Essential Mathematical Biology
2. *J. D. Murray*: Mathematical Biology Volume I and II
3. *H. R. Thieme*: Mathematics in Population Biology

MAJOR ELECTIVE II (MAT 215242)

Total Marks: 50   Credit Points: 4   Duration: 6 months

2. Differential Topology

1. Smooth mappings: Inverse Function Theorem. Local Submersion Theorem (Implicit Function Theorem).

2. Differentiable manifolds: Differentiable manifolds and submanifolds; examples, including surfaces, $\mathbb{S}^n$, $\mathbb{R}P^n$, $\mathbb{C}P^n$ and lens spaces; tangent bundles; Sard's Theorem and its applications; differentiable transversality; orientation.

3. Vector fields and differential forms: Integrating vector fields; degree of a map, Brouwer Fixed Point Theorem, No Retraction Theorem. Poincare-Hopf Theorem; differential forms, Stokes Theorem.

References

1. *Guillemin and Pollack*: Differential Topology, Prentice-Hall, 1974 (basic reference).
2. *Hirsch*: Differential Topology, Springer. 1976.
3. *Milnor*: Topology from the differential viewpoint. University of Virginia Press, 1965.
4. *Spivak*: Calculus on Manifolds, Benjamin, 1965

MINOR ELECTIVE II (MAT 215243)

Total Marks: 50 Credit Points: 4 Duration: 6 months

1. Fluid Dynamics

Introduction. Fluid Properties. Ideal Fluids. Viscous compressible and incompressible fluids. Non-Newtonian fluids. Theory of Stress and Rate of strain for fluids. Isotropic fluids. Stokes' hypothesis for Newtonian fluids.
Navier-Stokes'equation.
One dimensional inviscid incompressible flow.Euler's equation.Bernoulli equation.
Two dimensional and three dimensional inviscid incompressible flow. Basic Equations.E ulerian equations of motion. Circulation. Stokes'Theorem., Kelvin's theorem. Velocity potential and irrotational flow. Integration of equations of motion. Bernoulli's equation. Steady Motion. Stream function. Source and sink. Radial Row. Vortex flow. Doublet Motion of solid bodies in Fluid.
Laminar flow of Viscous incompressible fluid. Similarity of Flows. Reynold's number. Flow between parallel plates. Couette flow. Plane Poiseuille flow. Steady flow in pipes.
Boundary layer concept. Boundary layers in two dimensional flow. Boundary layer along a flat plate. The Blasius solution.
Inviscid compressible flow. Field equations. Circulation. Propagation of small disturbance. Sound waves. Steady isentropic motion. Mach number and cone. Bernoulli's equation. lrrotational motion. Velocity potential. Bernoulli's equation for unsteady flow. Steady channel flow. Mass flux through a converging channel. Flow through nozzle. Normal shock waves.
Surface waves. Basic equations. Boundary condition. Progressive waves. Group velocity.
Standing waves.

References

1. *L.M. Milne-Thomson*: Theoretical Hydrodynamics.
2. *P.K. Kundu and Iva M.Cohen*: Fluid Dynamics, Harcourt India.
3. *H. Lamb*: Hydrodynamics. Dover Publication.
4. *F. Chorlton*: Text Book of Fluid Dynamics. CBS Publ.
5. *H. Schlichting*: Boundary Layer Theory, McGraw Hill.

MINOR ELECTIVE II (MAT 215243)

Total Marks: 50   Credit Points: 4   Duration: 6 months

2. Algebraic Topology and Category Theory

Homotopy Theory: (Marks 20)
Homotopy and paths. Homotopy and Homotopy classes. Homotopy equivalences, Null homotopy, Relative homotopy, etc. Composite of homotopic spaces. Contractible spaces, deformation, strong deformation retraction etc. Path-connected spaces - their union, intersection and continuous images. Product and inverse of paths. Homotopy of paths and products of homotopic paths.

Covering spaces and covering maps
Covering spaces and covering maps. Properties of covering maps. Path lifting property and Homotopy lifting theorem.

Fundamental group
Definition and verification. Homomorphism and isomorphism of fundamental groups. Fundamental groups of Circle. Fundamental groups of some known surfaces, e.g. cylinder, punctured plane, torus, etc.

Homology Theory: (Marks :20)
Finite Simplicial Complexes
Simplicial complexes.Polyhedra and Triangulation. Simplicial approximation, barycentric subdivision and simplicial approximation theorem.

Simplicial Homology
Orientation of simplicial complexes. Simplicial chain complexes, boundaries and cycles, homology groups- some examples. Induced homomorphisms. Induced homology groups. Some applications for e.g. invariance of dimension, no retraction theorem, Brouwer's fixed point theorem, etc.

Category theory : (Marks -10)
Category
Definition and Examples.Objects- initial, terminal and null objects. Morphisms- epi, monic, isomorphism, section and retraction, uniqueness of identity morphism.

Functor
Covariant and contravariant functors. Definition and examples. Faithful and full functors. Equivalent Categories. Subcategory and full subcategory, Dual category and principle of duality. Natural Transformation - concept only with examples. Fundamental groups as a covariant functor between the category of pointed spaces and base-point preserving continuous maps and the category of groups and homomorphisms. Homology groups as a covariant functor between the category of all topological spaces and continuous maps and the category of groups and homomorphisms.

References

1. *A. Hatcher*: Algebraic Topology, Cambridge University Press
2. *W. Massey*: Algebraic Topology, Springer (GTM)
3. *M.J. Greenberg and J. R. Harper*: Algebraic Topology: A first course, Perseus books, Cambridge
4. *S. Deo*: Algebraic Topology: A Primer, Hindustan Book Agency (TRIM 27)
5. *S. Maclane*: Categories for the Working Mathematicians (second edition), Springer (GTM)